

A new electromagnetic mass model with non static conformal symmetry

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Abstract

We provide a new electromagnetic mass model admitting non static conformal symmetry. We conclude that the pressure and density failed to be regular at the origin but gravitational mass is always positive and vanishes in the limit $r \rightarrow 0$ i.e. it does not have to tolerate the problem of singularity. Further, we match interior metric with the exterior (Reissner-Nordström) metric and determine the values of the k and r_0 parameters expressed in function of mass, charge and radius of the spherically symmetric charged objects i.e. electron. We also find the Kretschmann scalar $K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ and check for singularity. In addition, the NEC and WEC energy conditions are discussed.

1 Introduction

In recent past, Herrera and Varela [1] discussed electron model by assuming the equation of state $p_{radial} + \rho = 0$ with an ad hoc anisotropy. Recently, Ray et al [2] have studied electron model admitting a one parameter group of conformal motion, and also a new electromagnetic mass model admitting Chaplygin gas equation of state with particular specialization was developed [3]. In this study, we extend both the works [1] and [2].

Valuable studies performed in the last two decades point out that a very interesting topic are the charged imperfect fluids spheres with a space-time geometry that admits a conformal symmetry, both in the static and in the generalized nonstatic cases [4], [5], [6], [7], [8], [9] (and references cited therein).

⁰Key words: Electromagnetic mass; Non static conformal symmetry; Einstein-Maxwell equations

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These classes of solutions are used in relativistic astrophysics for developing the star models. Herrera and his collaborators [5], [6], [7] searched for exact solutions of the field equations for static spheres considering that the static and spherically symmetric fluid geometry presents also a conformal symmetry. Ponce de Leon [8] gave regular static solutions with anisotropic pressure in the approach of a conformally flat sphere. Maartens and Maharaj [4] looked deeper and obtained regular solutions of the Einstein-Maxwell equations for charged imperfect fluids with conformal symmetry.

We investigate a new electromagnetic mass model admitting non static conformal symmetry. The paper is organized as follows: in Section 2 we give the basic equations which are the Einstein-Maxwell field equations combined with the electromagnetic tensor field with anisotropic fluid, and in Section 3 the solutions with some particular cases are given together with the graphical representation of the results. In Section 4 we match interior metric with the exterior (Reissner-Nordström) metric and find out k and r_0 parameters in terms of mass, charge and radius of the spherically symmetric charged objects i.e. electron. Section 5 is devoted to a discussion of the results.

2 Basic Equations for the New Electron Gas Model with Non Static Conformal Symmetry

For performing our study we consider the Einstein-Maxwell field equations combined with the electromagnetic tensor field with

anisotropic fluid, which together with an appropriate equation of state (EOS), $p_{radial} + \rho = 0$ yield the basic equations used for developing a new electron gas model with non static conformal symmetry.

The most general energy momentum tensor compatible with spherical symmetry is

$$T_{\nu}^{\mu} = (\rho + p_t)u^{\mu}u_{\nu} - p_t g_{\nu}^{\mu} + (p_r - p_t)\eta^{\mu}\eta_{\nu}, \quad (1)$$

with

$$u^{\mu}u_{\mu} = -\eta^{\mu}\eta_{\mu} = 1.$$

with ρ the matter density, and p_r and p_t are the radial pressure and the transverse pressure of the fluid, respectively.

The static spherically symmetric space-time is given by the line-element (in geometrized units with $G = 1 = c$)

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2)$$

where the functions of radial coordinate r , $\nu(r)$ and $\lambda(r)$ are the metric potentials.

For this metric, the Einstein Maxwell field equations are

$$e^{-\lambda} \left[\frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} = 8\pi\rho + E^2, \quad (3)$$

$$e^{-\lambda} \left[\frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = 8\pi p_r - E^2, \quad (4)$$

$$\frac{1}{2}e^{-\lambda} \left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda') \right] = 8\pi p_t + E^2. \quad (5)$$

where p_i , ρ and $E(r)$ represent fluid pressures (radial and transverse), matter-energy density and electric field, respectively.

The electric field is expressed

$$(r^2 E)' = 4\pi r^2 \sigma e^{\frac{\lambda}{2}}. \quad (6)$$

The equation (6) gives the following form for the electric field

$$E(r) = \frac{1}{r^2} \int_0^r 4\pi r^2 \sigma e^{\frac{\lambda}{2}} dr = \frac{q(r)}{r^2}, \quad (7)$$

with $q(r)$ the total charge of the sphere under consideration.

Like most of the researchers studied electron model, we assume the equation of state

$$p_r = -\rho, \quad (8)$$

The assumption (8) is consistent with the 'Causality Condition' $|\frac{dp}{d\rho}| \leq 1$ [10]. It is very common to the Cosmologists to use the matter distribution obeying this type of equation of state with equation of state parameter $\omega = -1$ (known as false vacuum or ρ - vacuum) to explain acceleration phase of the Universe [11].

3 Solutions

Let us now consider the problem of charged fluid sphere under conformal motion through CKV which can be given by [we follow the reference [4]]

$$L_\xi g_{ij} = g_{ij;k} \xi^k + g_{kj} \xi^k_{;i} + g_{ik} \xi^k_{;j} = \psi g_{ij}, \quad (9)$$

where L represents the Lie derivative operator, ξ is the four vector along which the derivative is taken, ψ is the conformal killing vector, and g_{ij} are the metric potentials [9].

For a vanishing ψ the equation above yields the Killing vector, the case $\psi = \text{const.}$ corresponds to the homothetic vector, and for $\psi = \psi(x, t)$ we obtain conformal vectors. In this way CKV allow a complete study of the spacetime geometry and, moreover if $\psi = 0$ the considered space-time is conformally flat, the Weyl tensor also is zero, and these conformally flat space-times correspond to gravitational fields without sources of matter that can produce them.

The proposed charged fluid (electromagnetic mass) space time is mapped conformally onto itself along the direction ξ .

Here, one takes ξ is non static but ψ is static as

$$\xi = \alpha(t, r) \partial_t + \beta(t, r) \partial_r, \quad (10)$$

$$\psi = \psi(r) \quad (11)$$

The above equations give the following set of expressions [4]

$$\alpha = A + \frac{1}{2}kt, \quad (12)$$

$$\beta = \frac{1}{2}B r e^{-\frac{\lambda}{2}}, \quad (13)$$

$$\psi = B e^{-\frac{\lambda}{2}}, \quad (14)$$

$$e^\nu = C^2 r^2 \exp \left[-2k B^{-1} \int \frac{e^{\frac{\lambda}{2}}}{r} dr \right], \quad (15)$$

where C, k, A, B are constants. According to [4] one can set, $A = 0$ and $B = 1$ by re-scaling.

The equation of state (8) implies

$$\nu = -\lambda. \quad (16)$$

Equations (15) and (16) yield

$$-\lambda' = \frac{2}{r} - 2k \frac{e^{\frac{\lambda}{2}}}{r}. \quad (17)$$

Solving this equation, one gets,

$$e^{\frac{\lambda}{2}} = \frac{1}{\left(k - \frac{r}{r_0}\right)}, \quad (18)$$

where r_0 is an integration constant.

We discuss the model by assuming the following assumption.

$$\sigma e^{\frac{\lambda}{2}} = \sigma_0 r^s, \quad (19)$$

where σ , σ_0 , λ and s represent the charge density of the spherical distribution, charge density at the center of the system, metric potential and a constant, respectively. Usually, the term $\sigma e^{\frac{\lambda}{2}}$ inside the integral sign in equation (7) is known as volume charge density. One can interpret the assumption (19) as the volume charge density being polynomial function of r [12].

Thus finally, we obtain the following set of solutions for different parameters

$$e^\lambda = e^{-\nu} = \frac{1}{\left(k - \frac{r}{r_0}\right)^2}, \quad (20)$$

$$\psi = \left(k - \frac{r}{r_0}\right), \quad (21)$$

$$E(r) = \left[\frac{4\pi\sigma_0}{(s+3)}\right] r^{s+1}, \quad (22)$$

$$q(r) = \left[\frac{4\pi\sigma_0}{(s+3)}\right] r^{s+3}, \quad (23)$$

$$8\pi\rho = \left(k - \frac{r}{r_0}\right)^2 \left[\frac{2}{r(kr_0-r)} - \frac{1}{r^2}\right] + \frac{1}{r^2} - \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}, \quad (24)$$

$$8\pi p_r = \left(k - \frac{r}{r_0}\right)^2 \left[\frac{1}{r^2} - \frac{2}{r(kr_0-r)}\right] - \frac{1}{r^2} + \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}, \quad (25)$$

$$8\pi p_t = \frac{1}{r_0^2} \left[3 - \frac{2(kr_0-r)}{r}\right] - \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}. \quad (26)$$

The mass of the electron can be obtained as

$$M = \int_0^r 4\pi r^2 \left[\rho + \frac{E^2}{8\pi}\right] dr = \frac{k}{r_0} r^2 + (1-k^2) \frac{r}{2} - \left(1 + \frac{1}{2}k^2\right) \frac{r^3}{3r_0^2}. \quad (27)$$

We point out that the metric potentials $\nu(r)$ and $\lambda(r)$ and ψ depend on k , r_0 and r coordinate, the electric field $E(r)$ and the electric charge $q(r)$ present a dependence in function of σ_0 , s and r , and the matter energy density ρ , the radial pressure p_r and the transverse pressure of the fluid p_t depend on k , r_0 , s and r . For the mass of the electron we obtain a dependence on k , r_0 and r . In Fig.1 we plot the variations of ψ vs r . In Fig.2 we have the plot for the variations of the $\nu(r)$ and $\lambda(r)$ metric potentials against r .

In Fig.3, Fig.4, Fig.5, Fig.6, Fig.7 and Fig.8 we plot the electric charge $q(r)$, the electric field $E(r)$, the matter energy density ρ , the radial pressure p_r , the mass of the electron M and the transverse pressure of the fluid p_t against the r parameter, respectively.

We have $\rho = -p_r$ which implies that a positive value of the matter energy density ρ determines a negative value of the radial pressure p_r .

Further, the energy condition $\rho > 0$ is satisfied for $\left(k - \frac{r}{r_0}\right)^2 \left[\frac{2}{r(kr_0-r)} - \frac{1}{r^2}\right] + \frac{1}{r^2} > \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}$. This means that there exists a limiting value of the radial coordinate for the violation of WEC condition. The NEC energy condition $\rho + p_r \geq 0$ is respected only in the limiting case (for equality with zero). The violation of NEC implies the breakdown of causality in general relativity and the violation of the second law of thermodynamics [13]. The condition $\rho + p_t > 0$ is satisfied for $\left(k - \frac{r}{r_0}\right)^2 \left[\frac{2}{r(kr_0-r)} - \frac{1}{r^2}\right] + \frac{1}{r^2} + \frac{1}{r_0^2} \left[3 - \frac{2(kr_0-r)}{r}\right] > \left[\frac{32\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}$

Thus the energy conditions are satisfied with some particularizations.

One can note that there is no singularity at $r = 0$ for the metric coefficients, σ , E if $s > -1$. The pressure and density failed to be regular at the origin but gravitational mass is always positive and will vanish as $r \rightarrow 0$ i.e. it does not have to tolerate the problem of singularity.

We also find the Kretschmann scalar

$$K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{4 \left(10r^2k^2r_0^2 - 12r^3kr_0 + 6r^4 + r_0^4 - 2r_0^4k^2 + 4r_0^3kr - 2r_0^2r^2 + k^4r_0^4 - 4k^3r_0^3r \right)}{r^4r_0^4}$$

and this expression diverge for $r = 0$. The Kretschmann scalar becomes finite $K = \frac{24}{r_0^4}$ for $r \rightarrow \infty$ combined with a nonvanishing value for r_0 .

4 Matching with Reissner Nordström metric

To match interior metric with the exterior (Reissner-Nordström) metric, we impose only the continuity of g_{tt} , g_{rr} and $\frac{\partial g_{tt}}{\partial r}$, across a surface, S at $r = a$

$$1 - \frac{2m}{a} + \frac{Q^2}{a^2} = \left(k - \frac{a}{r_0}\right)^2, \quad (28)$$

$$\frac{m}{a^2} - \frac{Q^2}{a^3} = -\frac{1}{r_0} \left(k - \frac{a}{r_0}\right). \quad (29)$$

From, the above three equations, one could find the values of the unknowns k and r_0 in terms of mass, charge and radius of the spherically symmetric charged objects i.e. electron.

$$r_0 = \pm \frac{\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right]^{\frac{1}{2}}}{\left(-\frac{m}{a^2} + \frac{Q^2}{a^3}\right)}, \quad (30)$$

$$k = \pm \frac{\left[1 - \frac{3m}{a} + \frac{2Q^2}{a^2}\right]^{\frac{1}{2}}}{\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right]^{\frac{1}{2}}}. \quad (31)$$

Note that \pm signs make r_0 to be positive.

5 Discussions

A new electromagnetic mass model admitting non static conformal symmetry is investigated and in this view we performed our calculations with the Einstein-Maxwell field equations combined with the electromagnetic tensor field of an anisotropic fluid

and with an appropriate equation of state (EOS), $p_{radial} + \rho = 0$. For describing the behaviour of our model we assume that $\sigma e^{\frac{\lambda}{2}} = \sigma_0 r^s$ and obtain the expressions for the metric potentials $\nu(r)$ and $\lambda(r)$, ψ , the electric field $E(r)$, the electric charge $q(r)$, the matter energy density ρ , the radial pressure p_r , the transverse pressure of the fluid p_t , and the mass of the electron M . The pressure and density diverge for $r \rightarrow 0$, and is interesting to find out how this singularity could be overcome. The mass of the electron vanishes for $r = 0$. We extend our study and match interior metric with the exterior (Reissner-Nordström) metric, and also establish the values of the k and r_0 parameters and we find their dependence on mass, charge and radius of the spherically symmetric charged objects i.e. electron. Moreover, the NEC and WEC energy condition are discussed. We notice that $\rho + p_r = 0$ and the NEC energy condition is satisfied only in the limiting case. For checking the WEC energy condition we also verify $\rho \geq 0$ and $\rho + p_t \geq 0$. We conclude that the all energy conditions are satisfied. The Kretschmann scalar $K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ presents a singularity for $r \rightarrow 0$ and becomes finite in the limit $r \rightarrow \infty$ combined with a nonvanishing value for r_0 . In the present work, we have used non static conformal symmetry technique to search electron like micro particle. It may be interesting to extrapolate the present investigation to the astrophysical bodies, specially quark or strange stars.

6 Acknowledgements

FR is thankful to Jadavpur University for financial support. We are also grateful to Dr. S. Ray for several Illuminating discussions.

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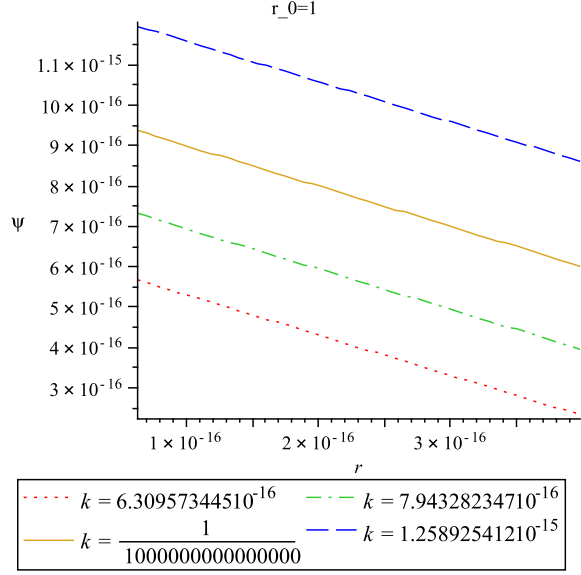


Figure 1: The diagram of the conformal factor with respect to radial coordinate 'r' for different k.

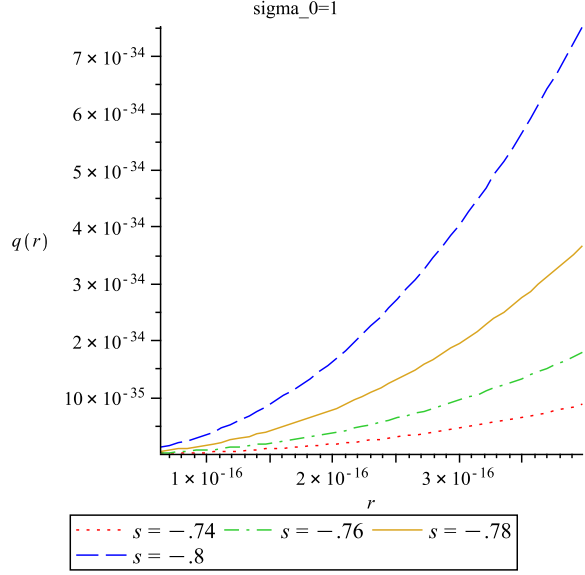


Figure 3: The diagram of the Electric charge with respect to radial coordinate 'r' for different s.

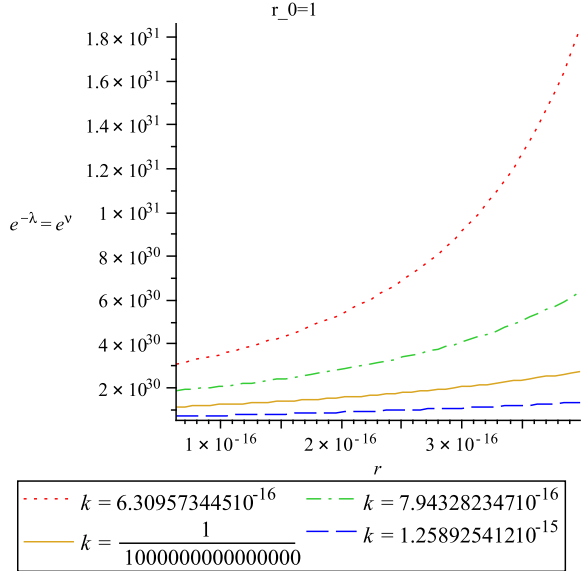


Figure 2: The diagram of $e^\nu (= e^{-\lambda})$ with respect to radial coordinate 'r' for different k.

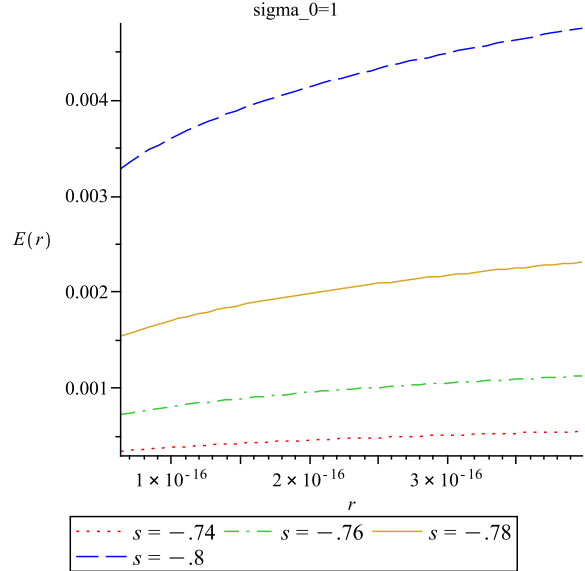


Figure 4: The diagram of the Electric field strength with respect to radial coordinate 'r' for different s.

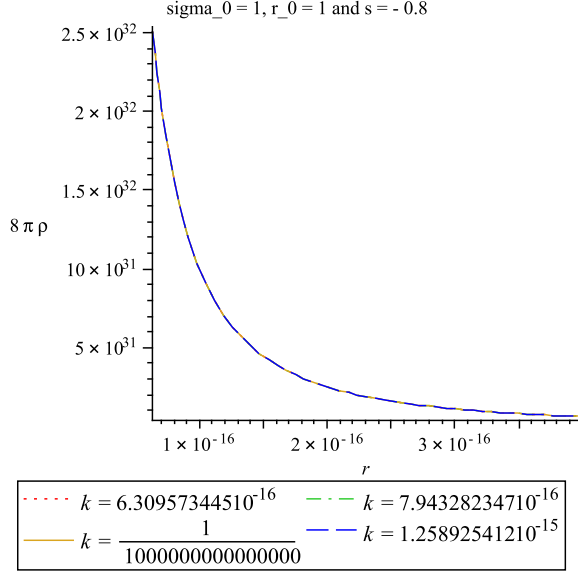


Figure 5: The diagram of the energy density with respect to radial coordinate 'r' for different k.

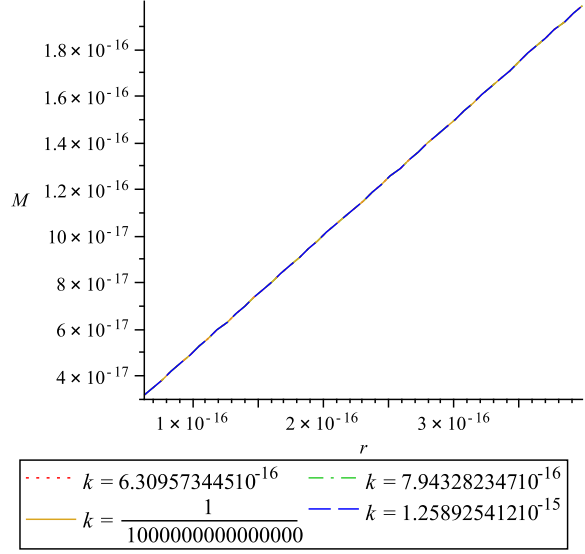


Figure 7: The diagram of the mass function with respect to radial coordinate 'r' for different k.

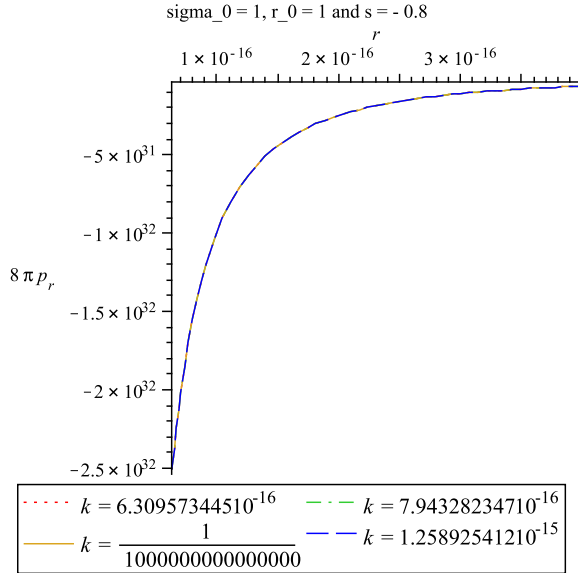


Figure 6: The diagram of the radial pressure with respect to radial coordinate 'r' for different k.

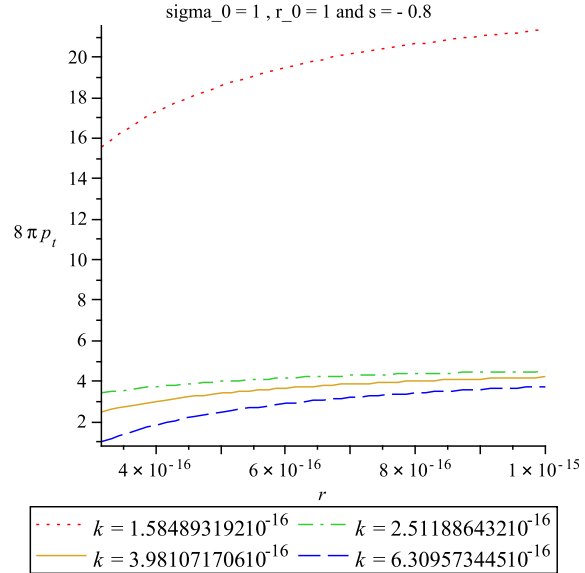


Figure 8: The diagram of the transverse pressure with respect to radial coordinate 'r' for different k.